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ABSTRACT

A developmental curriculum project is summarized which investigates the feasibility of using the PLATO computer assisted instruction (CAI) system to assist in the presentation of material in a geometry course which requires pictorial responses by the student. Geometrical concepts such as congruence, symmetry, perpendicularity, parallel, triangle, rectangle, etc., are discussed prior to consideration of the features needed for computer control of the student's tracing experiences. Methods are described by which students can draw patterns at their terminals; the computer's judging pattern and teaching logic are also outlined. Evaluations of student acceptance of the program and of the degree to which the program achieves its goals with inexperienced students indicate that the CAI system can assume routine functions involving pictorial student responses. The major advantages are that the CAI system keeps accurate student records, permits frequent program revisions, and allows the instructor to give individual attention to members of large groups without unduly delaying the progress of any single student. (PB)

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TEACHING SELECTED GEOMETRY TOPICS VIA A COMPUTER SYSTEM

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TEACHING SELECTED GEOMETRY TOPICS VIA A COMPUTER SYSTEM

John Richard Dennis, Ph.D.
Department of Education
University of Illinois, 1968

NOTE: This is an abbreviated version of a doctoral dissertation prepared under the direction of Professor Max Beberman of the University of Illinois, whose help is gratefully acknowledged.

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INTRODUCTION

For the past four years the staff of the University of Illinois Committee on School Mathematics (UICSM) has been developing an informal geometry course for junior high school students. (1) One objective of this curriculum effort was to develop a set of materials through which students could explore geometry and its ideas, could conduct simple experiments to verify or refute conjectures, could formulate broad generalizations and explore their consequences.

Briefly, these materials take the isometries of a plane as their mathematical basis from which to explore the usual properties of plane geometry. These mappings are introduced and studied with the aid of a translucent paper called a tracing sheet. The tracing sheet provides a model for congruence matchings of various figures. By describing three rather specific motions for a tracing we also have a model for the three basic isometries - translations, rotations, and reflections. Through these tracing sheet motions, some properties of isometries are studied. Then, based upon these properties, students conduct other tracing experiments that lead them to the more traditional ideas about the various triangles and quadrilaterals, perpendicularity, parallelism, etc.

This informal geometry course has been revised several times, and has been tried with several groups of students in Philadelphia, Memphis, Los Angeles, Honolulu, and New York City. The results of these trials have

(1) Sanders, Walter J. and Dennis J. Richard, "Congruence Geometry for Junior High School," The Mathematics Teacher, 1968, Vol. 61, pp. 354-369

suggested other revisions and methods of presentation that may make the use of this type of material more efficient. Much time is spent in the course in studying and experimenting with each of the classes of isometries. It is well known in mathematics that one type of isometry, the reflection, can be used to generate all the others. So, in relation to this informal work, the following question seems appropriate.

> Would it not save time to study just reflections in early stages of the course?

The authors had imagined the course presented in a laboratory setting, with a minimum of time spent in lecturing. In this type of setting, the teacher moves from student to student, or from group to group, offering individual help and suggestions. In all classes conducted in this mode it was noted that a not insignificant portion of the students' time was spent waiting for the teacher to verify answers to exercises. Many of these answers took the form of pictures drawn to certain specifications. These observations caused me to ask another question about the course:

> If put under computer control, could the use of such material be aided, and if so, what sacrifices in other phases of the course would be necessary?

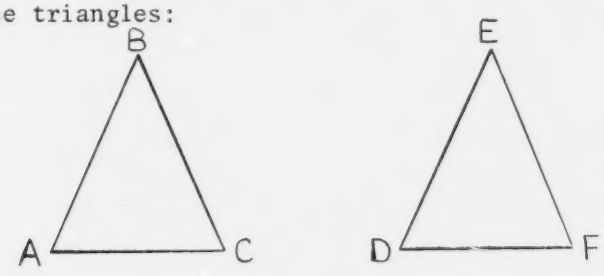
These are the considerations which led to the present project. This report briefly describes the subject matter and the features needed for computer control.

SUBJECT MATTER APPROACH

The first idea that needs to be communicated to the student is the meaning of the phrase <u>exactly alike</u> and the relationship this phrase has to a tracing sheet. By "exactly alike" for plane figures we shall mean

that a tracing of one of them can be made to match the other exactly. This match need not be achieved in any particular position or orientation. It may be possible to achieve a matching in several ways. It must be possible to achieve a matching in at least one way. When it is possible to match exactly a tracing of one figure with a second figure, we say that the figures are congruent, and the matching is called a congruence.

By experimenting with a tracing, congruences can be separated into two types - face up congruences and face down congruences. The distinction is an easy one. For a face down congruence, the tracing sheet must be turned over to make the figures match. Having made this distinction we concentrate on the effects of these two types of congruences. To do this, it is easiest to look at a pair of figures for which there are both face up and face down congruences, and to focus attention more narrowly upon parts of the figures, rather than the entire figures. For example, consider these triangles:



There are two ways to match a tracing of triangle ABC with triangle DEF, one face up and one face down. To focus our attention more sharply we ask questions like:

> For each congruence, what part of triangle DEF is matched with the tracing of segment AB?

Such questions have many variations for points, segments, angles, etc.

This gives an introduction to the notion of corresponding parts for a congruence, and, of course, since the tracing is used to match these parts, corresponding parts under a congruence are congruent.

The next step in this development is to apply the notions of congruence and corresponding parts to a single figure rather than two different figures. Specifically, we ask how many self-congruences, either face up or face down, a figure has. For face down self-congruences a very important observation comes to light. For each face down self-congruence of a figure, there is a line each of whose points corresponds with itself. It is such lines which we shall call lines of symmetry.

After some practice at recognizing figures which have lines of symmetry, finding the total number of lines of symmetry for a figure, and examining the corresponding parts for the face down congruences associated with the lines of symmetry, the next step is to begin to classify triangles and quadrilaterals on the basis of their lines of symmetry. For triangles, there are those with no lines of symmetry, those with one line of symmetry, and those with three lines of symmetry. There are no triangles with exactly two lines of symmetry. Such properties as a pair of congruent sides and a pair of congruent angles for each line of symmetry are easily introduced. Once the properties have been throughly investigated by the student, one can introduce the standard names scalene, isosceles, and equilateral for the three types of triangles.

Before classifying quadrilaterals, it is convenient to first call

attention to one particular type of face up self-congruence. This is the half-turn self-congruence. For each half-turn self-congruence, there is one point of the plane which corresponds with itself. This point is called a center of symmetry or a point of symmetry.

This is also a convenient time to introduce the ideas leading to perpendicular and parallel lines. For each two lines in a plane, sometimes one is a line of symmetry for the other. When this happens the lines make "square corners" with each other and the angles formed are all congruent. So we say that whenever one line is a line of symmetry for a second line, the lines are called perpendicular lines.

To get at parallel lines, we look at several pairs of lines and ask which pairs of lines have a line of symmetry in common. Those pairs of lines which do have a common line of symmetry are called parallel lines.

Both the parallel and the perpendicular definitions lend themselves to many interesting tracing experiments which focus attention on some fundamental issues in geometry. One of these is the problem of picturing unbounded sets like lines. The key question for the student is:

> What does it mean to say that a tracing of a line matches that line?

The students soon see that by this one does not mean complete superposition of the tracing as in the case of figures like triangles, but merely mean that the tracing lines up along the line traced. With these considerations out of the way, quadrilaterals are easily classified according to their symmetries. For quadrilaterals, lines of symmetry have a property that was not encountered when studying triangles.

A line of symmetry for a quadrilateral need not go through a vertex of the figure but, if it does, it goes through two vertices. So, for quadrilaterals, we have diagonal and nondiagonal symmetry lines.

The following table shows the results of the quadrilateral classification in the order they are treated.

point symmetry, diagonals cross	parallelogram
a pair of parallel sides, diagonals cross	trapezoid
1 nondiagonal symmetry or a point of symmetry, diagonals cross	isosceles trapezoid
1 diagonal symmetry, diagonals cross	kite
2 diagonal symmetries, point symmetry	rhombus
2 nondiagonal symmetries, point symmetry, diagonals cross	rectangle
2 diagonal and 2 nondiagonal symmetries, point symmetry	square

As in the case of triangles, the usual properties about congruent and parallel sides, congruent angles, bisecting and perpendicular diagonals, etc., follow from the fact that corresponding parts for the various self-congruences are congruent.

Following is a flow-chart summary of how these mathematical ideas are presented. Many other ideas of geometry can be approached in a like manner.

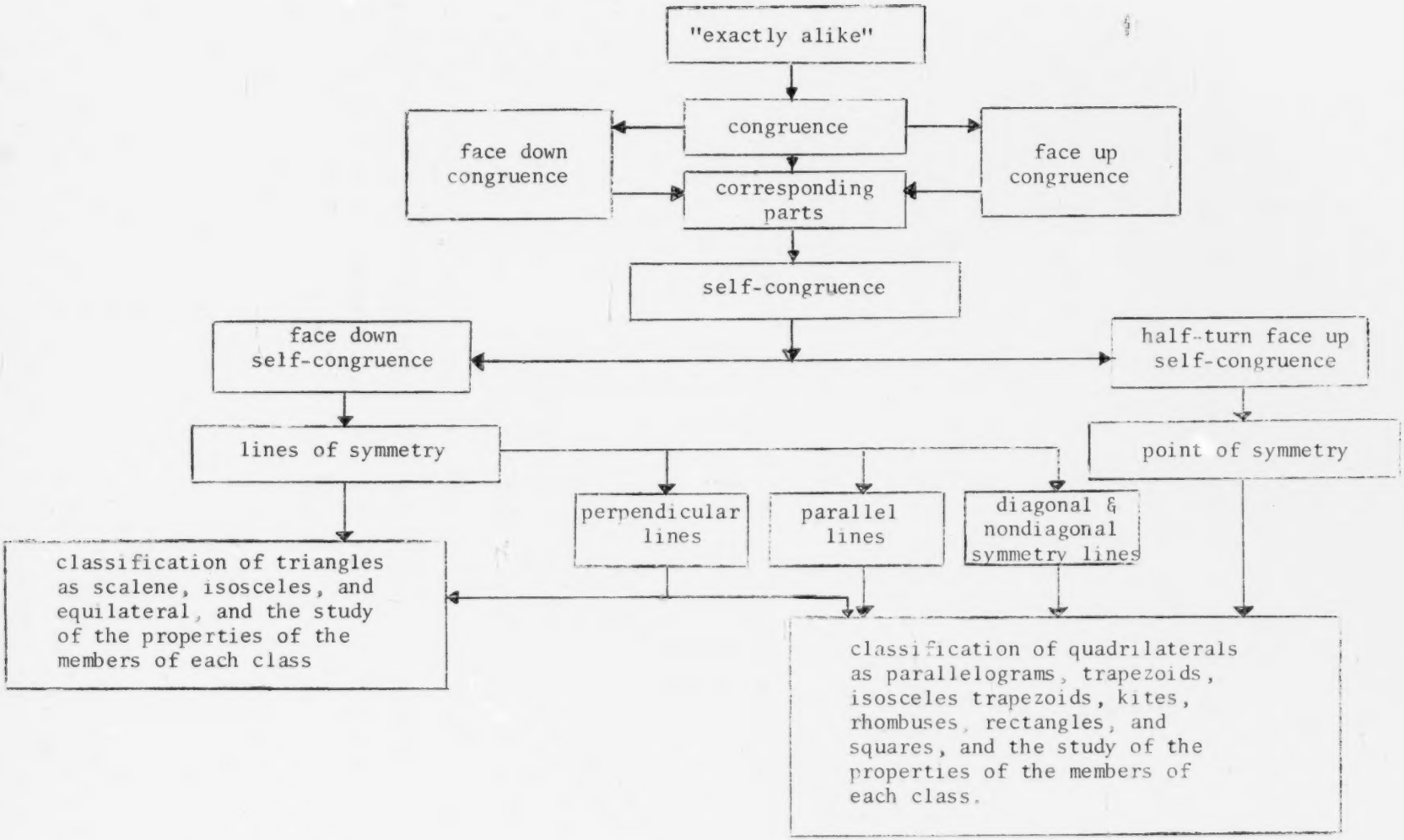

"exactly alike"
congruence
face down congruence
face up congruence
corresponding parts
self-congruence
face down self-congruence
half-turn face up self-congruence
lines of symmetry
point of symmetry
perpendicular lines
parallel lines
diagonal & nondiagonal symmetry lines
classification of triangles as scalene, isosceles, and equilateral, and the study of the properties of the members of each class
classification of quadrilaterals as parallelograms, trapezoids, isosceles trapezoids, kites, rhombuses, rectangles, and squares, and the study of the properties of the members of each class.

FIGURE 1

Flow Chart of Mathematical Development

COMPUTER CONTROL

As mentioned earlier, another matter of concern in this study is the feasibility of putting this type of informal instruction under the control of a computer-based instruction system. Two problems immediately arise.

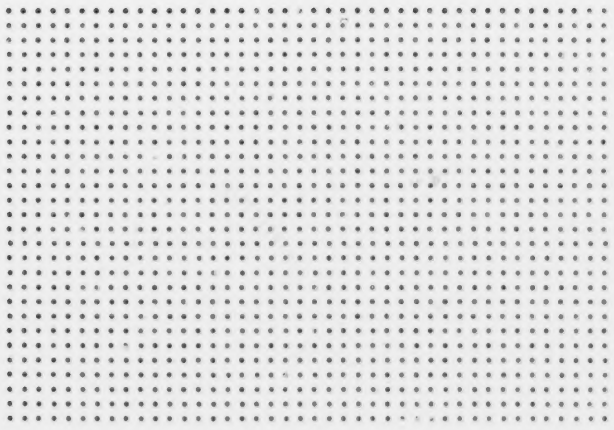
1. How to coordinate tracing experiments with computer controlled lessons.
2. How to provide picture drawing experience in computer controlled lessons.

The first of these was easy to treat. The lessons were accompanied by a workbook containing plates. Each plate has drawings that are either like those shown on the TV screen for a particular unit (frame) or are referred to by the questions of a particular unit.

The second problem was a little more difficult. The solution to it is described below.

Picture Construction

To make it possible for students to draw pictures on the TV screen, it was necessary to write computer programs that would, on command of the student, mark points and draw segments. The first step was to devise a means for selecting points on the screen. For this purpose I used a rectangular lattice of dots (see figure on following page.)



The dots of this lattice are approximately 1/8 inch apart. The lattice is plotted in the center of the TV screen. The students use the lattice as they would use a piece of lattice paper. They select dots of the lattice; then they instruct the computer to mark these dots and to connect them with segments. For the selection process I have defined eight keys (upper case) of the keyboard which allow the student to move a bright spot around the lattice in any of eight directions. These keys are arranged in a skewed rectangular pattern on the keyboard, as shown in the diagram:

The arrows on each key denote the direction in which depressing that key moves the bright spot.

When the student moves the bright spot to a point he wishes to use in a picture, he must communicate his selection to the computer. To do this, a key labeled mark has been defined on the keyboard. When the student pushes this key, the computer records the last location of the bright spot. The student is then ready to select and mark another point. When he marks his second point, the computer draws the segment determined by the first and second choices. If the student marks a third point, the computer draws the segment determined by the second and third points; etc.

To construct a triangle, or any closed polygonal figure, it is necessary for the student to mark as many points as there are vertices in the figure he intends to draw. Having done this, the computer has drawn one less than the desired number of segments. The segment from the last marked vertex to the first has not been drawn. To do this another key, called the close key, has been defined.

These keys may be used in several different ways. For n-gons the procedure is to mark n points and then to close. To draw a segment the student need only mark two points (no close is necessary). He may also be asked to mark individual points or any combination of points and segments.

Just as the student can draw figures on the screen, so the teacher (author) may construct figures as a part of the initial display in any particular unit. The format for this follows the process used by the students. If the teacher wishes a triangle to be drawn, he gives the

computer three vertices and a close order. If the close order is not given, an angle will be drawn with the second listed point as vertex. Segments and isolated points are included in a like manner.

Picture Evaluation

Once the student has constructed a pictorial response, he requests evaluation. When given this command, the computer must compare the student's response against those which the author has told it to accept. One standard method of telling the computer what to accept has been to give it a list of all acceptable responses, and then to rely upon its speed to search for a match with the student's response.

In the case of pictorial responses, this method of providing acceptable responses leaves much to be desired. You will recall that pictures are drawn on a lattice. Now imagine an exercise like:

Draw a rectangle.

and consider the task of providing the computer with all acceptable responses to this item. Although the computer could undoubtedly make a very rapid search of such a list, if it had it, the real problem is the time needed to prepare a list for several such exercises and the amount of computer memory needed to store them. Needless to say, this approach was not pursued.

Instead of approaching this problem by the "method of exhaustion", I chose to first make a list of all types of exercises that I might want to give to the student. Such a list looks something like this:

1. Make a scalene, isosceles, equilateral, and/or right triangle.
2. Make a kite, parallelogram, rhombus, rectangle, square, trapezoid, or isosceles trapezoid.
3. Make a line parallel (perpendicular) to a given one.
4. Make a figure congruent to a given one.
5. Mark points of a particular line.
6. Draw certain lines (such as symmetry lines, perpendicular bisectors, angle bisectors, etc.).

For each of these types all correct responses have a very definite and predictable pattern. So, I constructed a judging program that would read and evaluate the pattern of a student's response. The end product of this effort is several small programs each of which does one of the following:

1. Check a figure for a pair of congruent sides
2. Check a figure for a right angle.
3. Check a figure for one or more pairs of parallel sides.
4. Compute the slope of a segment for parallel (perpendicular) checks.
5. Check two segments (one given, one constructed) for congruence.
6. Check of correct number of marked points (and noncollinearity in the case of triangle or quadrilateral).
7. Check a constructed figure for congruence with a given figure or previously constructed figure.
8. Compute segment lengths.

These programs may be called individually, or in sequence, depending upon the type of pattern one intends to judge. In addition to these there is a main branching program which interprets the answer code given it by the author, activates the appropriate subprograms to do

the checking, and interprets their findings, taking further action when instructed by the author to do so.

The code which instructs this judger on the type figure to look for is constructed in four parts:

1. Number of points that must be marked for a correct answer.
2. Type of check to perform.
3. Specific points of the lattice that must be included in the answer, or that determine a line of reference for the answer (as in the case of a parallel or perpendicular to a given line).
4. Specific side lengths to look for in the answer.

For the second of these the author merely includes the appropriate number from the following list:

1 = isosceles triangle

2 = equilateral triangle

3 = right angle

4 = isosceles right triangle

5 = specific line (such as symmetry line)

6 = line parallel to a given one

7 = line perpendicular to a given one

8 = point of a certain line

9 = (blank)

10 = parallelogram

11 = rectangle

12 = rhombus

13 = square

14 = kite

15 = trapezoid

16 = isosceles trapezoid

17 = (blank)

18 = (blank)

19 = (blank)

20 = general congruence with a given figure

21 = general congruence with the previously constructed figure

This list may be extended at any time new options are desired, and the blank options may also be used for additional features. (For the present study students are not asked to draw equilateral triangles on the screen. The plotting screen is not continuous, so it is impossible to make an exactly equilateral triangle. An alternative would be to allow nearly equilateral triangles in some tolerance range. This too was rejected for the present study. The lattice on which the students draw pictures has its dots far enough apart to cause visual distortion in a triangle made as nearly equilateral as that lattice allows).

Most of the checks performed by this pattern judger are based upon standard analytic formulas for slope, segment length, etc. In order to explain in detail the assumptions behind the various geometry checks that may be made, and the manner in which these checks are made, it is first necessary to explain how the computer has been

instructed to store information about a student's picture.

Since the pictures are composed of straight line segments put end-to-end, the computer only needs to record the end points of these segments. However, the computer needs some coordinatization for the array of points used in pictures. As mentioned earlier only the dots of the lattice are accessible to the students. So, for referencing by the computer (and also by the subject matter author), the rows of this lattice are numbered 1 through 28 and the columns 1 through 42. For each point marked by the student, the computer records its row and column of the lattice. (However, the computer first translates the coordinates into the actual coordinates relative to the TV screen before storing, since these numbers are more readily used for plotting).

For successive vertices of a polygon these coordinates are stored in successive words of a list in the computer's memory. Each word of this list is made up of 48 bits. In addition to the coordinates of the point, the location of the preceding point is stored in the word, as is the location of the word holding the next point (if there is one). A typical list for a quadrilateral might look like this:

	X COORDINATE OF POINT	Y COORDINATE OF POINT	NOTHING	LOCATION OF PREVIOUS POINT	LOCATION OF NEXT POINT	IDENTIFYING TAG
	12 bits	12 bits	6 bits	6 bits	6 bits	6 bits
1st word	80	65		4	2	
2nd word	130	75		1	3	
3rd word	155	100		2	4	
4th word	95	125		3	1	

The rightmost 6 bits hold an identifying tag which allows the computer to decide whether the word holds coordinates for a point of a picture or codes for letters of a verbal response. This list of computer words is used for both types of response since, for a given exercise, the words will only be used in one of these ways. Also notice that the previous point number of the first word is 4, and the next point number of the fourth word is 1. These numbers appear this way only when a CLOSE command has been given by the student (or the subject matter author).

In each such list the segments are determined by adjacent words. The first segment is determined by the first and second words, the second segment by the second and third words, ..., the n - 1st segment by the n - 1st and nth words, the nth segment by the nth word and the first word (if the close order was given).

With the coordinates of the vertices stored in this manner, the first check performed is to see if the required number of points have been marked, and, in the case of a triangle or quadrilateral, to see if any three successive points are collinear.

If this test is passed then the various geometry checks are performed as follows:

1. isosceles triangle-check first and second segments for congruence; if congruent, triangle is isosceles; if not congruent, check second and third segments for congruence; if congruent, triangle is isosceles; if not congruent, check third and first segments for congruence; if congruent, triangle is isosceles; if not congruent, triangle is not isosceles.

2. equilateral triangle - check first and second segments for congruence; if not congruent, triangle is not equilateral; if congruent, check second and third sides for congruence; if congruent, triangle is equilateral; if not congruent, triangle is not equilateral.

3. right triangle - check first and second segments for perpendicularity; if perpendicular, triangle is right; if not perpendicular, check second and third segments for perpendicularity; if perpendicular, triangle is right; if not perpendicular, check third and first segments for perpendicularity; if perpendicular, triangle is right; if not, triangle is not right.

4. isosceles right triangle - see if triangle is isosceles; if not, triangle is not right isosceles; if isosceles, check to see if it has a right angle; if it does, it is a right isosceles triangle, if it has no right angle, it is not a right isosceles triangle.

5. specific line - see if line specified by the author contains each of the points marked by the student.

6. parallel line - see if line specified by the author has the same slope as that determined by the two points marked by the student. (For vertical lines, only the first coordinates are compared for equality.)

7. perpendicular line - see if the product of the slope of the line specified by the author and the slope of the line determined by the student's points is -1. (In case of horizontal or vertical lines this multiplication is not carried out. Instead the appropriate coordinates are checked for equality).

8. Point of a line - see if the point marked by the student is on the line specified by the author.

9. (blank)

10. parallelogram - check first and third segments for parallelism; if not parallel, figure is not a parallelogram; if parallel, check second and fourth segments for parallelism; if not parallel, figure is not a parallelogram; if parallel, figure is a parallelogram.

11. rectangle - see if there are two adjacent perpendicular segments; if not, figure is not a rectangle; if so, see if figure is a parallelogram, if not, figure is not a rectangle; if so, figure is a rectangle.

12. rhombus - see if first and second segments are congruent; if not, figure is not a rhombus; if so, see if figure is a parallelogram; if not, figure is not a rhombus; if so, figure is a rhombus.

13. square - see if first and second segments are perpendicular; if not, figure is not a square; if so, see if figure is a rhombus; if not, figure is not a square; if so, figure is a square.

14. kite - check first and second segments for congruence, if congruent, check third and fourth segments for congruence, if third and fourth segments congruent, figure is a kite; if third and fourth segments not congruent, figure is not a kite; if first and second segments not congruent, check second and third segments for congruence, if not congruent, figure is not a kite, if congruent, check first and fourth segments for congruence; if not congruent, figure is not a kite; if congruent, figure is a kite.

15. trapezoid - check first and third segments for parallelism; if parallel, figure is a trapezoid; if not parallel, check second and fourth segments for parallelism; if parallel, figure is a trapezoid; if not, figure is not a trapezoid.

    (Note: this definition accepts each of the following as a trapezoid:

    The third of these, the "bow-tie", was not eliminated because it was not expected that students used for this study would be drawing such figures.)

16. isosceles trapezoid - checks for a pair of opposite congruent sides; if not found, figure is not an isosceles trapezoid; if found, checks to see if figure is trapezoid; if not, figure is not an isosceles trapezoid; if so, figure is isosceles trapezoid.

    (Note: this procedure accepts the following as isosceles trapezoids.)

17-19. (blank)

20-21. general congruence - for these checks the computer has two lists of coordinates, one for the given figure and one for the constructed figure. The first segment of the given list is compared with each segment of the constructed list until a congruent one is found. (If none is found, the check ends and the figures are not congruent.) For example, suppose that the first segment of the given list is congruent to the fourth segment of the constructed list.

Given list	Constructed list
1st segment	1st segment
2nd segment	:
3rd segment	:
.	3rd segment
.	4th segment
.	5th segment
nth segment	:
	:
	nth segment

Next an orientation is established in the constructed list by checking to see whether the second given segment is congruent to the 5th or 3rd constructed segments. Let us say that the 2nd given segment is congruent to the 5th constructed segment. The next check is to see if the diagonal between the noncommon vertices of the 1st and 2nd given segments is congruent to the diagonal between the noncommon vertices of the 4th and 5th constructed segments. If this test passes then the 3rd given segment is compared with the 6th constructed one, and the diagonal between the noncommon vertices of the 2nd and 3rd given segments is compared with the diagonal between the noncommon vertices of the 5th and 6th constructed ones. The check continues in this manner, alternately checking sides and diagonals, until either the lists are completely matched or a noncongruent pair is found.

When a noncongruent pair is found before the list has been completely matched, the check reverts

to comparing the first given segment to the 5th, 6th, etc. constructed segments to see if there is another potential matching of the lists. It is important to know at each step of this procedure exactly where the check is being performed relative to each list. This is because in most cases, the checking in the constructed list will have to be jumped from the bottom (top) of that list to the top (bottom). This transition is made easy by having the locations of the next and previous points stored with the coordinates of each point.

It is interesting to note that this same procedure could be followed to write a general similarity checker for polygons. The first pair of segments would establish a ratio, and then successive checks would procede through the lists just as for congruence, except the ratios would be compared.

The Teaching Logic

Preliminary attempts at programming lesson material in which student could give pictorial responses were only partially satisfactory. These programs were tried during the summer of 1967 with a group of Upward Bound students. From the students' point of view the results of the trials were quite satisfactory, but from the author's point of view there was much to be desired. The biggest problem was the time required to prepare a lesson. It took about one month of nearly full time effort to prepare an hour long lesson.

Fortunately, a new tutorial teaching logic which offered a much more efficient format for preparing lessons was being prepared at that time by Paul Tenczar, a member of the Computer-based Education

Research Laboratory (CERL) of the University of Illinois. I took this logic and incorporated my picture drawing and pattern judging routines into it. The resulting teaching logic, used for this study, allowed me to prepare lessons at a very accelerated rate. In fact, for the 15 lessons used in this study I averaged 1½ hours of coding time to prepare a lesson for reading into the computer, and 2 hours per lesson of on-line editing and viewing.

The resulting logic has another feature which, in my opinion, is indispensable for computer-based instruction. It is possible to put several lessons into the computer memory simultaneously, and to have a student execute any of these at will. This provision kept each student busy throughout each class session, and yet allowed slower students to go at a pace that was comfortable to them. Many of the faster students take advantage of this option by reviewing previous lessons at the end of a class session.

EVALUATION

Evaluation can be carried out in several ways depending upon the context in which it is being conducted. Evaluation might entail a comparison of two things which are purported to accomplish the same task. In such a context, some questions of interest in the evaluation might be:

1. Which accomplishes the task more thoroughly?
2. Which is more economical or efficient?
3. If the poorer of the two is presently in use, what would be the consequences of a change to the better one?

In another context, however, it may be that those things being evaluated are not comparable to anything then in existence. In this type of context one is more interested in questions like:

1. What were those things intended to do?
2. What do they really do?
3. How well do the results of question 2 meet the expectations of question 1?

The context of the word reported here is more like the second of the above. We are interested in teaching informal geometry to junior high school students. It should be clear from earlier sections of this report that by "informal geometry" we mean to include more than vocabulary and a superficial classification of shapes. We would like the students to learn what it is about the various shapes that accounts for their overall appearance. Also, we would like the student to

approach these differences from different points of view. In the present school programs there is nothing with which to compare such material. The UICSM has recently been experimenting with informal geometry based upon motions, but its material has not been used in classes that would offer a basis for comparison with my approach.

The approach to the subject matter grows out of a study of symmetry. Again, symmetry is not commonly found in present day mathematics programs. Friend and Suppes (2) have written a programmed text for the elementary school in which the ideas of symmetry are used as a basis for developing properties of groups. Although similar at the beginning, the Friend-Suppes material very quickly branches in a different direction. The UICSM material can be cited here also, since symmetry is dealt with at great length. However, in this case, the route which I take to develop the symmetry ideas that will be used as foundation for the informal geometry explorations is different. The UICSM course is based upon all of the isometries, mine upon reflections, and although mathematically similar, my approach does not depend as heavily on a specific motion involved, but rather strives to emphasize the pairings (correspondences) that result.

The material of this study is being presented via a computer-based education system. This means, of course, that the format of

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[2] Friend, Jamesine and Suppes, Patrick, Symmetry. Palo Alto: Institute for Mathematical Studies of the Social Sciences, 1965.

the students' text is that of programmed instruction. Varied attempts have been made at teaching topics via programmed materials. The most consistent finding has been that programmed instruction produces results as good as instruction by a live teacher, but in much less time. So, it is reasonable to expect that if the material developed for this project were to be taught to similar groups of children by a live teacher, more time would be needed.

This project falls into the category of initial curriculum development. This in part determines the nature of the evaluation that is appropriate. In developing new presentations of subject matter, one decides that he wishes to convey certain information to his subjects and he prepares materials that in his belief will do the job. He then presents this material to students. As the presentation progresses, he observes his students and the way they react to and operate with the concepts involved. He also frequently asks questions which, if his material has done its job, the students should be able to answer. Based on the student's response to these questions the curriculum developer gets certain hunches. These fall generally in two categories:

1. Things are going well.
2. Things are not going well.

If the latter is the case then the author takes another look as his subject matter from the specific point of view of the questions the students were unable to answer and the incorrect responses which were given. This second look at the subject matter suggests revisions that

can be made to overcome the difficulties the students encountered. These changes are made and the process is repeated until the students exhibit the type of interactions with the chosen topics that the author desires.

The procedure used in this study is to present the subject matter to five successive groups of students. Each group is small (5-7 students) so that the author can observe the students as they execute the lessons. Questions are asked mainly by the computer, but if a student encounters trouble and asks for help, other questions will be asked by the instructor in an effort to ascertain the sources of the difficulty. The computer keeps a record of what each student does in executing the program. If this record keeping has been carefully designed by the experimenter then he can, in many cases, guess the thoughts of the student with a fair degree of accuracy. Questions asked by the instructor help further to sharpen these guesses. To keep records on the questions and responses made in person, the subject matter program has a unit (frame) accessible to both instructor and student, in which comments and notes may be recorded. So as the instructor asks questions of the student, he can type notes that are recorded by the computer. Later, when these records are sorted according to lesson and student, these additional questions and comments appear together with the string of incorrect responses given to the computer's questions.

Another advantage of computer control in projects of this type is the frequency of possible revisions. For example, as students execute lessons, if they are allowed to go at their own rate, it is not very long until there is at least one lesson between the fastest and the slowest. This means that within any one trial, I can make and evaluate the results of revisions several times. I pay particular attention to the problems of the more rapid students. I then make the revisions in the lessons before the next class session in which that lesson will be needed by a slower student, and I pay particular attention to the slower students as they execute that same section of the program. This procedure allows me to make several attempts at unusually difficult problems within a given group of students, as well as between one group and the next. The effectiveness of both type of changes to the program is tested in subsequent trials also.

Presentation of the computer program and the subject matter program to students was undertaken in an effort to answer three general questions:

I. Will students accept the information presented?

II. Will the subject matter program execute efficiently for students who have little prior experience with the topic, namely, geometry?

III. Will the computer program, as designed, execute properly and efficiently for several students?

To facilitate the gathering of data on these questions, it is desirable to break each one into a set of more specific questions.

I. Achievement

The subject matter program was designed to teach several distinct but related sets of information:

1. Properties of triangles and their classification as scalene, isosceles, and equilateral.
2. Properties of quadrilaterals, and their classification as kites, trapezoids, rhombuses, etc.
3. Properties of line and point symmetry.
4. Relationships among items 1, 2, and 3 above.
5. Vocabulary pertinent to items 1-4 above.
6. Skill in constructing pictures to fit a given description.

In line with these intentions, the following questions should be considered:

1. How well do students learn the standard classifications of triangles and quadrilaterals?
2. What definitions do students associate with these classes of figures, the ones based upon symmetry, or the standard Euclidean ones, which are derived from symmetry properties?
3. How skillful do students become at doing a construction on the computer?
4. Do computer acquired construction skills transfer to a paper and pencil context?
5. When students encounter a particular construction for the first time, is it necessary to first give them a sample of such a figure to copy, or if given, say, a piece of lattice paper, will students sketch their own sample and copy it?

II. Execution of Subject Matter Program

An assumption made during the initial writing of the subject matter program pertains to the length of each lesson, and the relative frequency of questions requiring verbal response and questions

requiring pictorial responses. I guessed that a picture drawing exercise would take 2-3 minutes on the average, while verbal response questions would require less than this amount of time. So I attempted to maintain an average lesson length of 20-25 units (frames), with about $\frac{1}{3}$ of these of the picture-drawing variety. Therefore, an appropriate question to attempt to answer is:

> Is the hypothesized lesson structure appropriate with respect to time?

Other important questions concern the design of the inquiry options provided for the lessons. These include sample figures of various types, the dictionary, and suggestions for performing experiments with paper and tracings. In line with this the following questions are pertinent:

1. Does the dictionary include the necessary information?
2. How frequently do students ask the instructor for help that is not present in the program?
3. How frequently do students ask the instructor for help that <u>is</u> present in the program, but is not easily accessible to them at that point in time?

III. Execution of Computer Program

Although the teaching logic used in this study is basically a tutorial one, with provisions for some student inquiry, a major addition was made in an effort to allow students to draw polygons, line segments, and points on the TV screen, and to have the computer judge these figures for certain specified characteristics. Two questions that needed to be answered about these additions to the teaching logic are:

1. Has the complexity of the picture-drawing process been reduced to a level that allows the student to learn that process in a reasonable amount of time?
2. Does the method chosen for programming the computer to draw these pictures cause any unusual interruptions or delays during execution by several students?
3. Are the judging routines general enough to handle the many possible student reponses?

SUMMARY

As was mentioned in the opening comments on evaluation, this was an initial curriculum development project. I was also investigating the feasibility of using a computer-assisted instruction system to assist in the presentation of material which requires many pictorial responses by the student as well as verbal and numerical responses.

It is clear from this study that a computer system can control exercises requiring pictorial responses as easily as it does other exercises. In fact with pattern-type judging routines the exercises requiring a pictorial response are much easier for the author to write than those requiring verbal responses where there are several correct responses that must be anticipated. The average times required for the computer to process student requests are not unreasonable as compared with response times from other types of subject matter used with the PLATO system. Prospects for the future are even better since newer systems will undoubtedly have a capability to plot or erase individual screen points without distorting registration with those points already plotted. This will eliminate the need for many of the full screen erase and replot operations that it was necessary to include in the teaching logic for this project.

It is also clear from the data collected during the trials of the lessons that the picture drawing process is easy for the students to learn. After some initial revisions, students of later trials were

able to acquire the necessary proficiency with this process in lesson 2. At the same time they were gaining familiarity with some of the language used in subsequent lessons.

One of the author's motives in revising the lessons was to reduce the amount of routine help needed by the students to a level where it would be possible for an instructor to handle an average sized class of 30-35 students without seriously delaying any one of them. Although specific data on success in achieving this goal is not available, it is my opinion, based on the amount of routine help needed by 7 students and projected to a group 5 times this size, that the goal was a reasonable one and that the lessons could be presented to a larger group without the serious delays experienced when presenting similar material to students without the assistance of a computer system.

Of course, a question of prime importance in most educational experiments is what do the students learn? In this study we have assumed that the student's knowledge before studying these lessons is limited to recognition of some of the shapes involved and that their knowledge of properties of these shapes is practically nonexistent. Some students interviewed before beginning the lessons know that a square has "square corners". Properties of sides and diagonals, as well as symmetry properties, are not familiar to the students.

After studying the lessons, students are given a test via the computer and a paper-pencil test. The latter is particularly valuable

in demonstrating the wide variety of properties the students are able to state upon completion of the lessons. Students are also able to make accurate pencil sketches of various figures. The students' knowledge of necessary and sufficient conditions for a particular shape improved.

In general, the PLATO system is able to assume much of the routine presentation of geometry material like that described in this report, accommodate unusual instructional features such as pictorial responses and tracing experiments, and eliminate much of the waste time experienced by students in teacher controlled classes.

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